Globally Optimal Synthesis of Minimal Heat Exchanger Networks with Simultanous Equipment Design

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Optimal Heat Exchanger Design
(Gonçalves et al., 2016)

Globally Optimal Synthesis of
Minimal Heat Exchanger Networks
(Chang et al., submitted)

Globally Optimal Synthesis of
Minimal Heat Exchanger Networks
with Equipment Design

Optimal Heat Exchanger Design

- *Design variables (discrete):*

1. Tube diameter
2. Shell diameter
3. Number of tube passes
4. Tube length
5. Pitch ratio
6. Layout
7. Number of baffle

Written as functions of binary variables

↓

All the other model continuous variables can be written using the design variables, hence the binaries

↓

Final model: ILP

WE USE A SET TRIMMING APPROACH

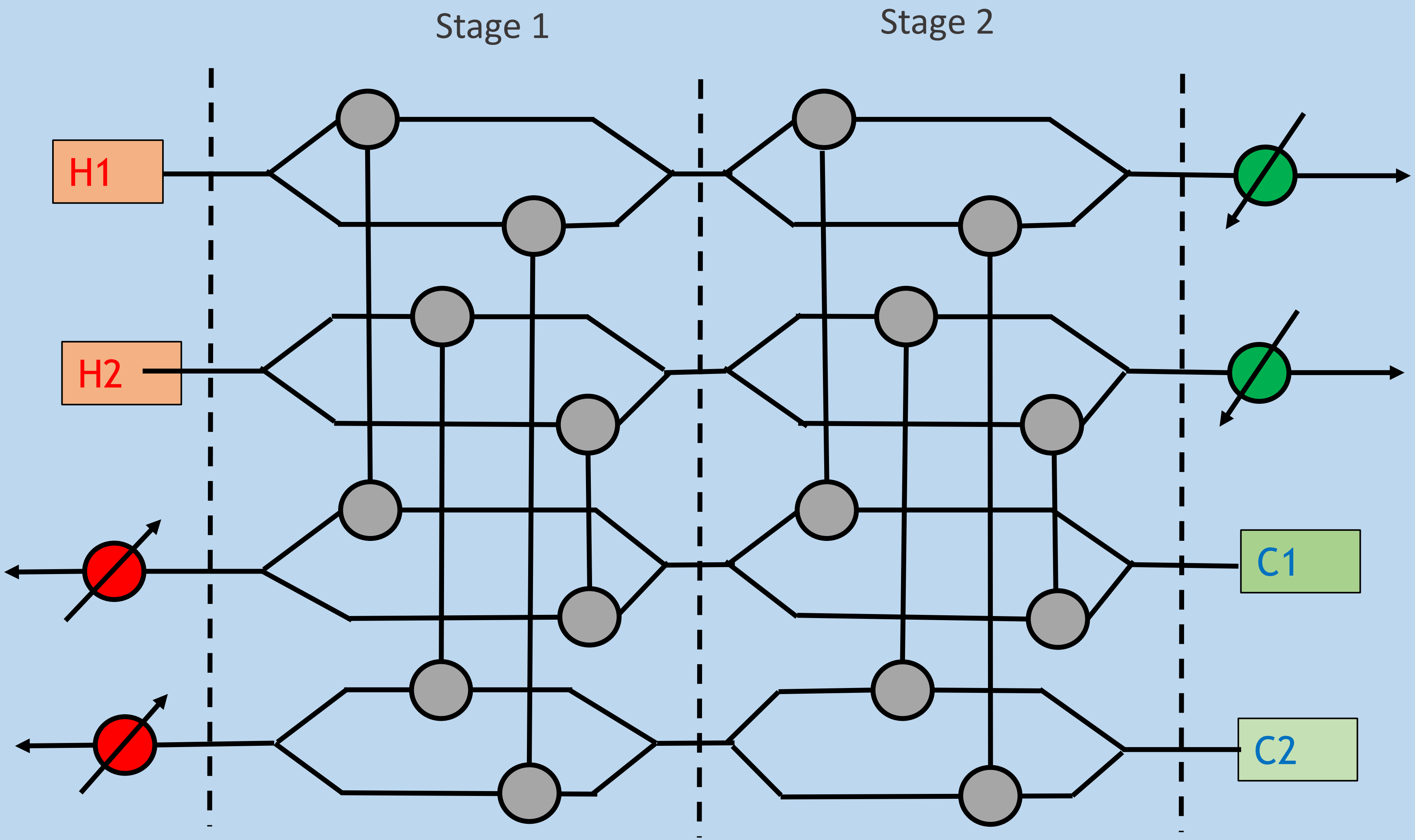
Heat Exchanger Network Synthesis

Hot streams need to be cooled

Cold streams need to be heated

How to match these streams?

Stagewise Superstructure (Yee and Grossmann, 1990)



Globally Optimal Synthesis of Minimal Heat Exchanger Networks

Models

PSTR
Chang et al.(2020)

Min α → Dummy objective function

s.t.

Synheat constraints (except for area equations)

$$EMAT = EMAT_{min}$$

$$\sum_{i \in HP} \sum_{j \in CP} \sum_{k \in st} z_{i,j,k} + \sum_{i \in HP} zcu_i + \sum_{j \in CP} zhu_j = N$$

→ Total number of matches equal to N=NH+NC+NU-1

This model finds a Minimal Structure (without Energy Loops)

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Models

PEmin/Pemax
Chang et al.(2020)

Min/Max E

s.t.

$$E = \sum_{j \in CP} qhu_j \qquad EMAT = EMAT_{min} \qquad \hat{E}_{min}^{user} \le E \le \hat{E}_{max}^{user}$$

$$\{z_{i,j,k} = 1, zhu_j = 1, zcu_i = 1\} \; \forall\, (i,j,k) \in MSTR_l \longrightarrow$$ Fixed matches

This model finds Minimum and Maximum Energy Consumption for a fixed Structure

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Models

PESTR
Chang et al.(2020)

Min α → Dummy objective function

Subject to

$$\hat{E} = \sum_{j \in CP} qhu_j$$ → Fixed energy

$$EMAT = EMAT_{min}$$

$$\{z_{i,j,k} = 1, zhu_j = 1, zcu_i = 1\} \; \forall \, (i,j,k) \in MSTR_l$$ → Fixed matches

This model finds the heat distribution for a given Energy consumption, EMAT minimum and a fixed Structure

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Models

PSTRR
Chang et al.(2020)

Min α ⟶ Dummy objective function

s.t.

$$EMAT = EMAT_{min}$$

$$\sum_{i \in HP} \sum_{j \in CP} \sum_{k \in st} z_{i,j,k} + \sum_{i \in HP} zcu_i + \sum_{j \in CP} zhu_j = N$$

$$\sum_{i,j,k \in MSTR_{l'}} \left(z_{i,j,k} + zcu_i + zhu_j\right) - \sum_{i,j,k \notin l'} \left(z_{i,j,k} + zcu_i + zhu_j\right) \leq card(MSTR_{l'}) - 1$$

As the MSTR model, this model finds a Minimal Structure (without Energy Loops) excluding previous results

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Smart Global Search Algorithm

Step 1 – Set *UBTAC* (best solution) = ∞

Step 2 – Solve *PSTR* to obtain a viable *MSTR*

Step 3 – For *MSTR*, solve *PEmin* – obtain E_{min}

Step 4 – For *MSTR*, solve *PEmax* – obtain E_{max}

Step 5 – Apply monotony test:

- *Solve PESTR for* $E=E_{min}$*. Evaluate TAC (*TAC_{min}*)*
- *Solve PESTR for* $E=1.01E_{min}$*. Evaluate TAC (*TAC_{min}^{+}*)*
- *Solve PESTR for* $E=E_{max}$*. Evaluate TAC (*TAC_{max}*)*
- *Solve PESTR for* $E=0.99E_{max}$*. Evaluate TAC (*TAC_{max}^{-}*)*

EACH EVAUATION IMPLIES

1) Obtaining areas
2) Calculate TAC

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Smart Global Search Algorithm

Step 6 – If $\{TAC_{min}^{+} - TAC_{min}\}\{TAC_{max} - TAC_{max}^{-}\} < 0$ – not monotone, go to **Step 8**

Step 7 – If $\{TAC_{min}^{+} - TAC_{min}\}\{TAC_{max} - TAC_{max}^{-}\} > 0$ – monotone

- If $\{TAC_{min}^{+} - TAC_{min}\} > 0$, $TAC = TAC_{min}$, go to **Step 9**
- If $\{TAC_{min}^{+} - TAC_{min}\} < 0$, $TAC = TAC_{max}$, go to **Step 9**

Step 8 – Apply Golden Ratio Search to obtain the best *TAC*. Use *PESTR* and then solve the heat exchanger design to each one of the matches, to obtain *TAC* for each point

Step 9 – If $TAC \leq UBTAC$, update *UBTAC*

Step 10 – Obtain another structure (*PSTRR*), if this is infeasible make *N=N-1*, if it continues to be infeasible go to **Step 11**, otherwise, go to **Step 3**

Step 11 – *UBTAC* is the global optimum

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Example data

Stream	T_{IN} (°C)	T_{OUT} (°C)	*Fcp* (kW/°C)	*h* (kW/m²°C)
H1	159.0	77.0	228.5	0.40
H2	267.0	88.0	20.4	0.30
H3	343.0	90.0	53.8	0.25
C1	26.0	127.0	93.3	0.15
C2	118.0	265.0	196.1	0.50
CU	20.0	40.0	-	0.53
HU	500.0	499.0	-	0.53

Heat exchanger cost: 25000 + 55 area ($/y)

Utility cost: 100 *qhu* + 10 *qcu* ($/kWy)

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Results

Example data

Heat exchanger design (Kern model)	
Design variable	*Number of alternative*
Shell diameter	0.203, 0.254, 0.305, 0.307, 0.387, 0.438, 0.489, 0.540, 0.591, 0.635, 0.686, 0.737, 0.787, 0.838, 0.889, 0.9398, 0.9906, 1.0668, 1.143, 1.2192, 1.295, 1.3716, 1.448, 1.524, 1.600, 1.676, 1.753, 1.829, 1.981, 2.134, 2.286, 2.438, 2.591, 2.743, 2.896, 3.048
Tube diameter	0.01905, 0.02540, 0.03175, 0.03810, 0.5080
Number of tube passes	1, 2, 4, 6
Pitch ratio	1.25, 1.33, 1.50
Layout	1 (square), 2 (triangular)
Length	1.2195, 1.8293, 2.4390, 3.0488, 3.6585, 4.8768, 6.0976
Number of baffles	1 – 20

$$\Delta P_{disp} = 100\ kPA$$

Physical properties
Cp = 2268 J/kg.K
ρ = 883 kg/m^3
$\mu = 3.55 \cdot 10^{-3}$ Pa.s
k = 0.1143 W/m.K

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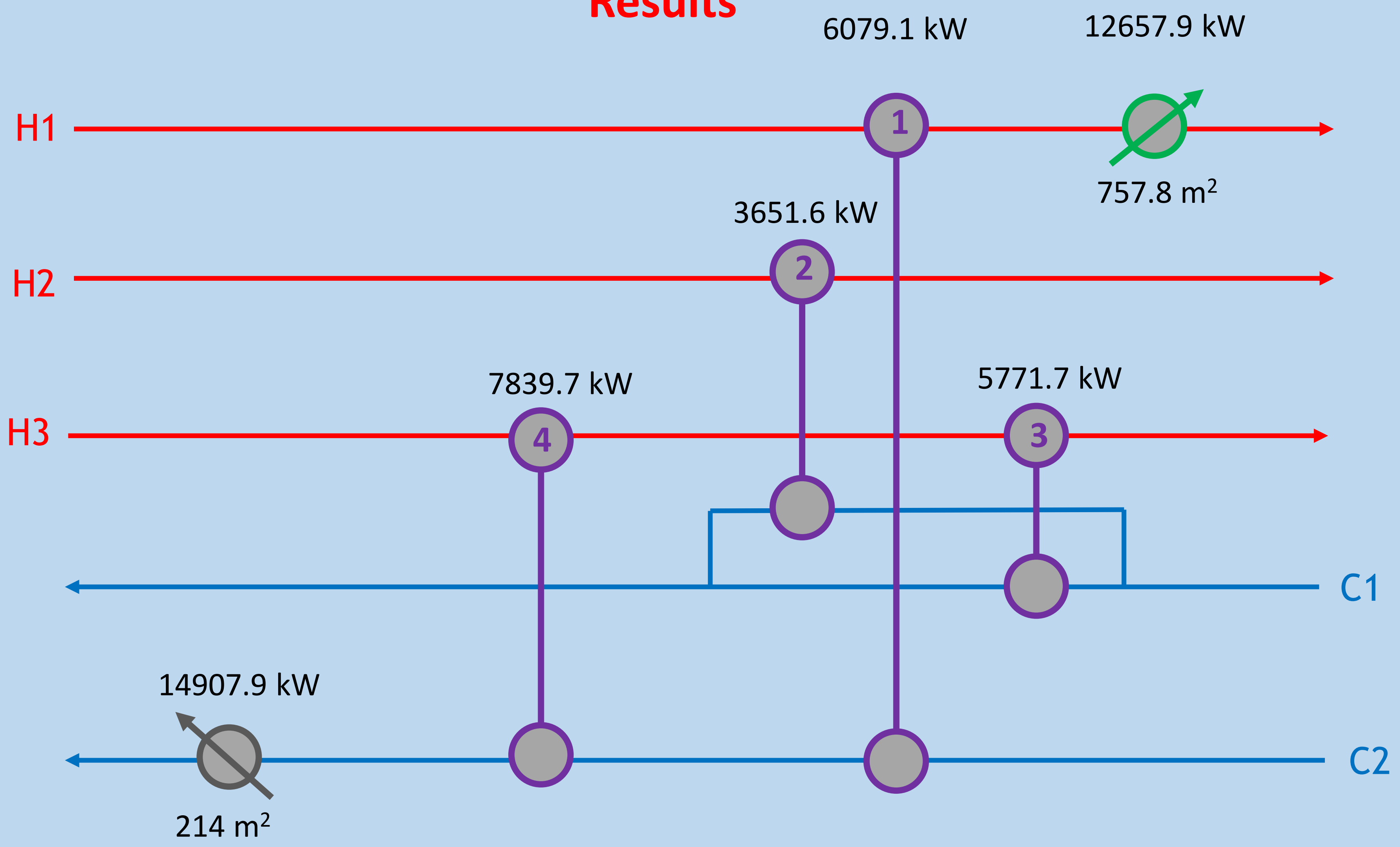
Results



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Results

	Heat Exchanger			
Variable	1	2	3	4
Ds	1.295	0.438	0.540	0.9398
dte	0.0254	0.01905	0.01905	0.01905
dti	0.0221	0.01575	0.01575	0.01575
Npt	6	4	4	4
rp	1.25	1.25	1.25	1.25
lay	2	2	2	2
L	4.8768	3.6585	4.8768	3.8565
Nb	17	20	20	16
Area	1585.4	120.9	245.2	278.3
ΔPt	96782	68564.7	95800.7	49123.3
ΔPs	65603.9	15132.7	43956.5	91458.9

TAC = 1,943,453 $/y

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Conclusions

- We are able to obtain a Globally Optimal Minimal Heat Exchanger Network with simultaneous HEX design
- Energy consumption varies from structures obtained using the constant U model (Chang et al., submitted)
- The sizes of the exchangers vary.
- THE STRUCTURE CAN VARY
- Other examples must be tested to finish this work